Transit-time method of optical stochastic cooling

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A transit-time method for stochastic cooling is extended and developed for optical stochastic cooling. Limitations on the damping times are analyzed. Illustrative applications of the method to the cooling of electrons, protons, and heavy ions are considered.

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I. INTRODUCTION

In Ref. [1] the utilization of a broadband optical amplifier was proposed for the stochastic cooling process in order to enhance the ultimate possibilities of the conventional microwave stochastic cooling technique [2]. It was shown that the radiation of a particle in a quadrupole wiggler can be amplified and applied back to the same particle in a dipole wiggler, producing therein an energy kick in proportion to the particle's transverse position in the quadrupole wiggler. It was also shown that this scheme is capable of providing damping for both transverse and longitudinal oscillations, but that cooling takes place only in the case of a small beam emittance $\leq 10^{-9}$ m. We show how to avoid this limitation by using a different approach.

II. DESCRIPTION OF THE WORKING PRINCIPLE

Typically, in conventional microwave stochastic cooling, a cooling system is comprised of a pickup, an amplifier, and a kicker [2]. Although optical stochastic cooling deals with the same cooling principle as the microwave stochastic cooling, all components of the cooling system mentioned above undergo substantial modifications. These modifications, which are associated with a transition into the optical frequency regime, will be highlighted below.

Consider an insertion in a storage ring designed for optical stochastic cooling, which includes two undulators, an optical amplifier, and a bypass. A schematic drawing of this insertion is shown in Fig. 1.

As shown elsewhere [3], the stochastic cooling method obeys the principle that, when passing the cooling system, each particle receives a correcting kick that is a superimposition of the *coherent* and *incoherent* components. The coherent component is responsible for damping, and is proportional to the deviation of the particle from the equilibrium momentum or the reference orbit. The incoherent component is due to the effects associated with other beam particles.

We first consider how the particle receives a coherent kick in the cooling insertion shown in Fig. 1. Moving along the insertion, the particle radiates an electromagnetic (EM) wave in the first undulator. This wave goes to the optical amplifier, while the particle follows the bypass trajectory and meets its amplified radiation in the second undulator. A subsequent interaction between the particle and the EM wave from its own radiation results in a change of the particle energy. The amount of the energy change depends on the phase of the EM wave of the radiation at which the particle enters the undulator.

The variation of this phase from particle to particle is due only to the particle traveling time in the bypass, since EM waves radiated by different particles propagate from the first undulator to the second undulator identically. Therefore, in order to have the energy change proportional to, for example, the particle momentum deviation:

First, adjust the propagation time of the EM wave and the traveling time of a particle with a zero momentum deviation such that this particle will enter the undulator at a phase with a zero electric field, and thus will not undergo any energy change.

Second, design the bypass optics such that particles with different momenta follow trajectories with different path lengths, so they will enter the second undulator with phase shifts (relative to the phase with zero electric field) proportional to their actual value of momentum deviation.

A similar approach is applicable to betatron motion.

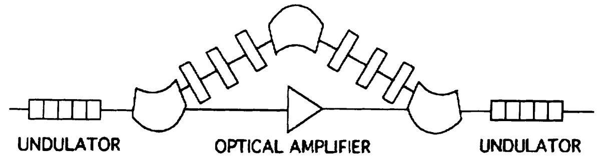

FIG. 1. Scheme of a cooling insertion in a storage ring.

What is required in this case is to link the particle traveling time in the bypass with some quantitative characteristic of the betatron motion (say, the betatron coordinate or angle, or a linear combination of them) at the beginning of the bypass, and to install undulators in a region with a nonzero dispersion function.

In the general case, particles can have a momentum deviation and a deviation in betatron coordinate and angle simultaneously. We now consider this case in more detail. Assume that at the entrance of the cooling insertion some arbitrary test particle has a momentum deviation $\delta_i = \Delta P_i/P$, a betatron coordinate x_i, and a betatron angle x'_i. Neglecting higher-order terms, the length of the trajectory of this particle in a bypass could be written $\ell_i = \ell_0 + x_i I_U + x'_i I_V + \delta_i(\eta_0 I_U + \eta'_0 I_V - I_D)$ [4], where ℓ_0 is the trajectory length of the reference particle having zero momentum deviation and zero betatron coordinate and angle, η_0 and η'_0 are the dispersion function and its derivative in the first undulator, and the symbols I_U, I_V, and I_D stand for integrals:

$$I_U = \int_L \frac{U(s)\,ds}{\rho(s)}, \quad I_V = \int_L \frac{V(s)\,ds}{\rho(s)}, \quad I_D = \int_L \frac{D(s)\,ds}{\rho(s)},$$

where $U(s)$ and $V(s)$ are two independent cosinelike and sinelike solutions of the homogeneous equation of the motion, ρ is the bending radius of the magnets, L denotes the position of the second undulator with respect to the first undulator, and $D(s)$ represents the contribution of the bypass magnets to the primary dispersion function.

In the first undulator, the test particle radiates an EM wave propagating in the z direction; $E_i = E_0 \sin(kz - \omega t + \phi_i)$ is the electric field of the EM wave with amplitude E_0 and phase ϕ_i, and $k = 2\pi/\lambda$ and $\omega = kc$ are the wave number and the frequency, respectively; $\lambda = [\lambda_u(1+K^2/2)]/2\gamma^2$ is the wavelength, c is the speed of light in a vacuum, γ is the Lorentz factor, λ_u is the undulator period, and K is the *undulator parameter* [5]. This radiation goes to the optical amplifier, while the particle follows the bypass and traverses it in a time $\Delta t_i = \ell_i/c$. The time Δt_0 required for radiation to pass all the way between undulators, including the amplifier delay, must be constrained and maintained by a feedback system to yield a condition $\ell_0 - c\Delta t_0 = \lambda/4$. Thus the particle arrives at the second undulator with a time delay $\delta(\Delta t) = \Delta t_i - \Delta t_0$ and with a phase shift $\Delta\phi_i$:

$$\begin{aligned}\Delta\phi_i &= k(\ell_i - \ell_0) \\ &= k[x_i\, I_U + x'_i\, I_V + \delta_i(\eta_0 I_U + \eta'_0 I_V - I_D)]\ , \quad (1)\end{aligned}$$

relative to the phase with zero electric field. In the second undulator, the particle interacts with the electric field of its own radiation and changes its momentum by

$$\delta P_i = g\frac{q\,E_0\,M\,\lambda_u\,K}{2c\,\gamma}\sin(\Delta\phi_i), \quad (2)$$

where q is the particle charge, M is the number of undulator periods, g is the amplification factor of the optical amplifier, and δP_i is the amount of the momentum change related to the coherent longitudinal kick $\Delta\delta_i = \delta P_i/P$.

In order to calculate a transverse kick from the energy kick, we define the dispersion function and its derivative in a location of the second undulator. For a lattice with a mirror symmetry relative to the central point of the bypass, these are η_0 and $-\eta'_0$, so that the changes of the particle betatron coordinate and angle at the exit of the second undulator are $\Delta x_i = -\eta_0(\delta P_i/P)$ and $\Delta x'_i = \eta'_0(\delta P_i/P)$.

Thus, after passing the entire cooling insertion, the test particle has received coherent longitudinal and transverse kicks that are proportional to a linear combination of the particle's momentum deviation and betatron deviations. We will see in the next section that a proper choice of the parameters of the bypass lattice makes it possible to use these kicks to simultaneously damp transverse and longitudinal oscillations.

The cooling technique described above has certain similarities with the transit-time method proposed in Ref. [6] for microwave stochastic cooling. This is why we have chosen to use the same name for this method, and refer to it as the *transit-time method of optical stochastic cooling.*

III. COOLING RATES

We have so far considered the interaction of the arbitrary test particle with the EM wave of its own radiation. However, each particle also interacts with the EM waves emitted by other particles moving behind it within a distance $\leq M\lambda$. These interactions constitute the incoherent component of the kick received by the particle. Assume that a test particle interacts with N_s electromagnetic waves (including its own wave) and consider again a change of the particle's momentum at the exit of the cooling insertion:

$$\delta_{ic} = \delta_i + G\,\sin(\Delta\phi_i) + G\sum_{k\neq i}^{N_s}\sin(\Delta\phi_i + \psi_{ik})\ . \quad (3)$$

Here, δ_{ic} is the relative momentum of the ith particle after the longitudinal kick, $\psi_{ik} = \phi_i - \phi_k$ and

$$G = g\frac{q\,E_0\,M\,\lambda_u\,K}{2c\,\gamma\,P}\ . \quad (4)$$

In the right-hand part of Eq. (3), the contribution of the test particle to the total kick is subtracted from the sum, so that the sum depicts only the incoherent component of the kick.

Expressions similar to Eq. (3) can also be written for transverse coordinates. We do this for a bypass with a mirror symmetrical lattice and with a $-I$ transfer matrix between undulators. In this case, $I_U = 0$, $I_V = 2D_0$, and $\eta_0 = D_0$, where $2D_0$ is a contribution to the value of the dispersion function in the second undulator coming solely from the elements of the bypass lattice. Thus, using the above definition of the transverse kick, we write

$$x_{ic} = x_i + D_0\, G\, \sin(\Delta\phi_i) + D_0\, G \sum_{k\neq i}^{N_s} \sin(\Delta\phi_i - \psi_{ik}) \ ,$$
$$x'_{ic} = x'_i - \eta'_0\, G\, \sin(\Delta\phi_i) + \eta'_0\, G \sum_{k\neq i}^{N_s} \sin(\Delta\phi_i + \psi_{ik}) \ . \qquad (5)$$

Here x_{ic} and x'_{ic} are the betatron coordinate and angle after correction.

We can now evaluate the averages of the quantities: $\Delta(\delta^2) = \delta_{ic}^2 - \delta_i^2$, $\Delta(x^2) = x_{ic}^2 - x_i^2$, and $\Delta(x'^2) = x_{ic}'^2 - x_i'^2$ taken over the ensemble of N_s particles and over the distribution of the particles in betatron coordinates and angles and momentum deviations. Then, using Eqs. (3) and (5) and applying the same technique as in Ref. [3], we can write for damping decrements:

$$\alpha_\delta = \frac{\overline{\Delta(\delta^2)}}{\overline{\delta^2}} = 2G(I_D - 2D_0\,\eta'_0)\, k \exp\left\{-\frac{\overline{\Delta\phi_i^2}}{2}\right\} - \frac{G^2 N_s}{2}\frac{1}{\sigma_\delta^2} \ ,$$
$$\alpha_x = \frac{1}{2}\left(\frac{\overline{\Delta(x^2)}}{\overline{x^2}} + \frac{\overline{\Delta(x'^2)}}{\overline{x'^2}}\right) = \frac{1}{2}\left[4G\, D_0\, \eta'_0\, k \exp\left\{-\frac{\overline{\Delta\phi_i^2}}{2}\right\} - \frac{G^2 N_s}{2}\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\frac{\beta}{\epsilon_x}\right] \ , \qquad (6)$$

where $\overline{\Delta\phi_i^2} = k^2[(2D_0)^2\overline{x'^2} + (2D_0\,\eta'_0 - I_D)^2\overline{\delta^2}]$ and substitutions $\overline{\delta^2} = \sigma_{p'}^2$, $\overline{x^2} = \epsilon_x\beta$, and $\overline{x'^2} = \epsilon_x/\beta$ are used, and where ϵ_x is the beam emittance and β is the beta function in the undulator. The exponential term appearance in the first terms of Eqs. (6) is due to the sinelike dependence of the coherent kick from the particle's phase shift. We can now define the optimal G by maximizing the sum of the decrements $\alpha_x + \alpha_\delta$ so that

$$G = \frac{2(I_D - D_0\,\eta'_0)\sigma_\delta^2\, k\ \exp\left\{-\overline{\Delta\phi_i^2}/2\right\}}{N_s\left[1 + \frac{\beta}{2\epsilon_x}\ \sigma_\delta^2\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\right]} \ , \qquad (7)$$

and

$$\alpha_x + \alpha_\delta = \frac{2(I_D - D_0\,\eta'_0)^2\ \sigma_\delta^2\left[k \exp\left\{-\overline{\Delta\psi_i^2}/2\right\}\right]^2}{N_s\left[1 + \frac{\beta}{2\epsilon_x}\sigma_\delta^2\left(\eta_0'^2 + \frac{D_0^2}{\beta^2}\right)\right]} \ . \qquad (8)$$

Consider at this point the limitations on the maximum attainable damping rates. First of all, $k\exp\{-\overline{\Delta\phi_i^2}/2\}$ reaches its maximum $k/\sqrt{e}$ (e is the base of the natural logarithm) at

$$\overline{\Delta\phi_i^2} = k^2\left[(2D_0)^2\frac{\epsilon_x}{\beta} + (2D_0\,\eta'_0 - I_D)^2\sigma_\delta^2\right] = 1 \ . \qquad (9)$$

Notice that the reduction of ϵ_x and σ_δ during the damping leads to a decrease of $\overline{\Delta\phi_i^2}$ and, correspondingly, to a decrease in the phase shifts of the individual particles. As a result, the coherent components of the particles' kicks are reduced, whereas the incoherent components of the kicks that do not depend upon ϵ_x and σ_δ remain the same. Since only coherent components of the kicks are responsible for damping, a decrease of $\overline{\Delta\phi_i^2}$ leads to a slowdown of the damping process. Fortunately, we can prevent this by keeping $\overline{\Delta\phi_i^2}$ at a constant level, independent from the current values of ϵ_x and σ_δ. According to Eq. (9), increasing $2D_0$ and $(2D_0\,\eta'_0 - I_D)$ will compensate for the reduction of ϵ_x and σ_δ. In order to do this, small adjustments in the bypass lattice during the damping process are required.

We can further simplify Eqs. (8) and (9) by assuming $(\eta'_0\sigma_\delta)^2 \approx \epsilon_x/\beta$ and $D_0^2 \ll \beta\eta_0'^2$. For example, for a case of equal decrements, when I_D is adjusted to be equal to $3D_0\,\eta'_0$,

$$G \approx \frac{\sigma_\delta}{\sqrt{e}\, N_s} \ , \qquad (10a)$$

$$\alpha_x = \alpha_\delta \approx \frac{1}{2e\, N_s} . \qquad (10b)$$

We see here that the number of passes through the cooling insertion required for a $1/e$ reduction of the beam emittance (and the beam energy spread in the second power) is equal to $2eN_s$. Recall that N_s is equal to the number of particles in the bunch within the distance $M\lambda$, if the transverse beam size in the undulator does not exceed the size of the transverse coherence of the EM waves. Otherwise,

$$N_s \simeq N\frac{M\lambda}{2F\ell_b} , \qquad (11)$$

where N is the number of particles in the bunch, ℓ_b is the bunch length, and F is ratio of the beam transverse area in the undulator to the transverse coherence area of the light.

For the next step, we introduce in formula (11) the bandwidth of the undulator radiation using the well known relation between the number of undulator periods and the width of the spectral line [full width at half maximum (FWHM)] on the first harmonic $\Gamma = \Delta\omega/\omega \approx 1/M$ [5]. Then, the damping time due to the optical stochastic cooling can be written

$$\frac{\tau_{x,\delta}}{T} = \frac{eN}{\Gamma}\frac{\lambda}{F\ell_b}\ , \tag{12}$$

where T is the revolution period (we assume only one cooling insertion in the ring).

Equation (12) was derived under the assumption that the bandwidth of the cooling system is defined by the bandwidth of the undulator radiation. However, the bandwidth of the undulator radiation can be made wider than the bandwidth of the optical amplifier. Therefore, in a more realistic case, the bandwidth of the optical amplifier should replace Γ in Eq. (12).

Although we have considered only the damping of horizontal betatron oscillations and energy oscillations, the analogous cooling technique could equally be used for damping the vertical oscillations. In the latter case, we need vertical dispersion in the undulators, and the orbit bend between undulators must be in the vertical plane.

IV. AMPLIFICATION FACTOR

In order to determine the amplification factor of the optical amplifier one can rewrite Eq. (10a) using G from Eq. (4):

$$g\frac{q\,E_0\,M\lambda_u\,K}{2c\gamma P} = \frac{\sigma_\delta}{\sqrt{e}\,N_s}. \tag{13}$$

In Eq. (13) the only unknown parameter besides the amplification factor is the amplitude of the electric field of the particle radiation E_0 . We evaluate E_0 in the waist of the light beam where the cross section of the coherent mode of the radiation (defined at the one σ level of the intensity distribution) is $A \approx 2\lambda\, M\lambda_u$. During one pass of the undulator with $K \approx 1$ (we assume the undulator with the maximum yield of the photons into the coherent mode) the particle emits into the coherent mode $\sim q^2/\hbar c$ photons of the energy $\sim \hbar\omega$. Therefore

$$\frac{c}{8\pi}E_0^2 A\Delta t_R = kq^2, \tag{14}$$

where $\Delta t_R = M\lambda/c$ is the duration of the radiation pulse. Using Eqs. (13) and (14) and substitution (12) for N_s, we finally find

$$g \approx \frac{1}{\sqrt{e}}\frac{\epsilon_{\|}\,\Gamma F}{r_0 N}\ , \tag{15}$$

where $r_0 = q^2/mc^2$ is the classical radius of the particle, m is the particle mass, and $\epsilon_{\|} = \gamma\,\ell_b\,\sigma_\delta/\sqrt{2\pi}$ is the invariant longitudinal emittance. Notice that $\epsilon_{\|}$ in Eq. (15) represents the current emittance at each moment of the damping process. Therefore a reduction of the amplification factor to follow the emittance reduction is required during the damping process (as well as the adjustments mentioned above in the bypass lattice). The equilibrium emittance is reached when all sources of damping are balanced by all sources of the emittance excitation. After that, the scheme remains stationary. If the optical stochastic cooling is the only source of damping, then we can define the absolute minimum emittance $\epsilon_{\|0}$, corresponding to the case where the only source of the emittance excitation is the radiation fluctuation in the undulators of the cooling scheme:

$$\epsilon_{\|0} = \sqrt{e}\frac{r_0 N}{\Gamma F}. \tag{16}$$

Notice that in this case $g = 1$.

V. POWER LIMIT

We have so far assumed that we are not limited by amplifier power. Although it is very likely for electrons, this might not be correct if we try to reach the optimal damping time, Eq. (12), working with protons and/or antiprotons and heavy ions. If, in order to reach the optimal damping, the required output power of the amplifier exceeds the available power, then the available power of the amplifier would determine the damping times:

$$\frac{\tau_{x,\delta}}{T} \approx \left\{\frac{N_{\rm tot}\lambda}{\overline{W}\,cT\,K^2}\frac{\sigma_\delta^2\,(E_b/q)^2}{Z_0}\right\}^{1/2} . \tag{17}$$

Here $\overline{W}$ is the available average output power of the optical amplifier, E_b is the equilibrium beam energy, Z_0 is the free-space impedance, and $K \le 1$. In deriving Eq. (17), we assume $\Gamma > 1/M$. Notice that $E_b/q \sim B\rho$, where $B\rho$ is the magnetic rigidity, does not depend on the particle charge and the atomic number.

VI. NOISE AND MIXING

There are two additional parameters usually associated with the stochastic cooling technique. These are the noise of the amplifier and the so-called mixing—a process of the re-randomization of the beam on the way from the kicker to the pickup.

The noise of the optical amplifier is due to spontaneous emission from the active medium, and is roughly equivalent to one noise photon in the value of the coherence at the amplifier front end. We should compare it with the noise we have already considered above: the $N_s\,q^2/\hbar c$ photons radiated by the N_s particles in the first undulator. Clearly, at this level, the noise of the amplifier is negligibly small.

As for mixing, the rule here is that the particles must not stay together with the same neighbors for more than one turn, since otherwise the incoherent heating will grow up [3]. It seems relatively easy to comply with this rule in the optical stochastic cooling. A complete rerandomization will occur if, during the passage from the second undulator to the first undulator, particles change positions inside the bunch on $\sim M\lambda$. Another possibility for randomization exists when the beam emittance is larger than $1/2\,k$. In this case, mixing in the transverse phase space will also occur.

As for the beam path from the first undulator to the second undulator, the rule is completely different. There must be no mixing here. It was realized in the scheme discussed above that in the linear approximation, there is no mixing. If the beam emittance is larger than $1/2\,k$, then second-order geometric and chromatic aberrations can significantly affect the synchronism between the particle and its radiation. One more precaution has to be taken in this case; namely, the optical system and the bypass lattice must possess identical focusing properties, including second-order geometric and chromatic aberrations.

VII. EXAMPLES

For purposes of illustration, we consider very schematically the application of optical stochastic cooling to three type of particles: electrons and/or positrons, protons and/or antiprotons, and heavy ions.

A. Electrons

Since electrons already have a good damping mechanism due to the synchrotron radiation, we examined what optical stochastic cooling can do in the low energy regime, where synchrotron radiation damping is weak. Therefore we considered a 150 MeV ring of a 60 m circumference having two cooling insertions: one for longitudinal-horizontal cooling and one for longitudinal-vertical cooling. We assumed typical beam parameters based on a positron beam after the conversion and acceleration to 150 MeV: $N = 5 \times 10^9$, normalized horizontal and vertical emittances of 5×10^{-4} m, bunch length $\ell_b = 2.5$ cm, and a relative energy spread of 1×10^{-3}. For the amplifier, we assumed a Ti:Al_2O_3 optical amplifier with a central wavelength of 0.8 μm and a bandwidth of 10%. With this set of parameters we calculated an optimal amplification factor of $g \approx 350$, the damping time for betatron oscillations of $\tau_{x,y} \approx 30$ ms, and the damping time for energy oscillations of $\tau_\delta \approx 15$ ms. With one bunch in the ring and with the amplification factor specified above, the average output power of the amplifier is about 5 W in each cooling insertion.

B. Protons

As an example, for the proton-antiproton machine we considered the collider TEVATRON (Fermilab). We assumed two cooling insertions—one for longitudinal-horizontal cooling and one for longitudinal-vertical cooling; six bunches with 1×10^{11} protons/bunch, a relative momentum spread of 3×10^{-4}, and a revolution frequency of 47.7 kHz. We also assumed a *dye* amplifier with an average output power of 100 W and a central wavelength of $\lambda = 0.8$ μm. The undulator radiation with this wavelength could be obtained in the undulator with a peak magnetic field of 8 T and $\lambda_u = 1.5$ m. For this set of parameters, we estimate damping times; we get ~ 5 min damping time for betatron oscillations and ~ 2.5 min damping time for synchrotron oscillations.

C. Heavy ions

In this example, we considered damping of lead ions ($Z = 82$) in the Super Proton Synchrotron (CERN) at an ion energy of 32.8 TeV. The following beam parameters, taken from the Large Hadron Collider design report [7], were used: 124 bunches of 1×10^8 ions/bunch, a relative momentum spread of 3×10^{-4}, and a revolution frequency of 43 kHz. Assuming two cooling insertions, an undulator with a peak magnetic field of 8 T, and $\lambda_u = 0.3$ m, and the same optical amplifier as above, we get ~ 2 min damping time for betatron oscillations and ~ 1 min damping time for synchrotron oscillations.

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